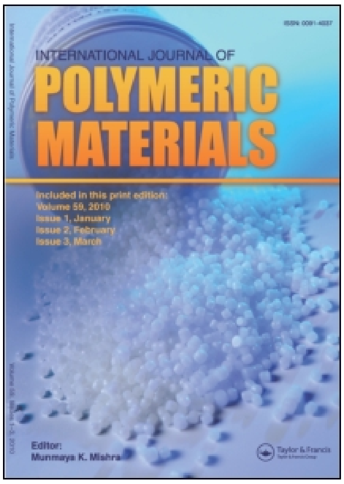


International Journal of Polymeric Materials
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713647664

Curves on the Fracture Surfaces of Brittle Amorphous Materials

F. Lednický[a]; Z. Pelzbauer[a]
[a] Institute of Macromolecular Chemistry, Czechoslovak Academy of Sciences, Prague 6, Czechoslovakia

To cite this Article Lednický, F. and Pelzbauer, Z.(1973) 'Curves on the Fracture Surfaces of Brittle Amorphous Materials', International Journal of Polymeric Materials, 2: 2, 149 — 165
To link to this Article: DOI: 10.1080/00914037308075306
URL: http://dx.doi.org/10.1080/00914037308075306

Intern. J. Polymeric Mater., 1973, Vol. 2, pp. 149–165

Curves on the Fracture Surfaces of Brittle Amorphous Materials†

F. LEDNICKÝ and Z. PELZBAUER

Institute of Macromolecular Chemistry, Czechoslovak Academy of Sciences Prague 6, Czechoslovakia

(*Received June* 11, 1972)

An equation has been derived describing possible shapes of the fracture curves which appear if two fracture fronts meet on the fracture surfaces of brittle amorphous materials. The characteristic forms of the curves are compared with the observed ones. The equation includes the following parameters: the centers distance of the primary and secondary fractures, the ratio of the spreading velocities of the fracture fronts originating in these centers, and the so-called activation distance, i.e. the distance of the primary fracture front from the center of the secondary front at a moment when the secondary fracture is activated. The fracture curves are divided into four groups according to the possible activation distance values. The shapes of the fracture curves described so far in the literature are shown to be special cases of the equation. The qualitative agreement between the theoretical and experimental fracture curves justifies the assumptions introduced while deriving the theoretical form. The determination of the parameter values can give new information on the fracture mechanism.

INTRODUCTION

When interpreting the lines on fracture surfaces it is assumed that the cracks in a brittle material propagate from two structural defects (fracture centers) in planes perpendicular to the direction of the acting force. If the distance between these planes is not too large, the material between the cracks is chipped out in a certain stage of destruction in such a way that the projection on the fracture surface thus obtained forms the so-called fracture curve.[1,2]

So far, various types of fracture curves have been detected on the fracture surfaces of the brittle materials (straight lines and open conic sections,[1–4,11]

† Presented at the conference "General Principles of Rheology", Prague, September 11–14, 1972.

closed curves,[4,8–10] the Wallner lines[5–7]), but only some of them have been interpreted in mathematical terms (straight lines, open and closed conic sections,[2–4] the Wallner lines[7]).

It has been the aim of the present paper to express theoretically the possible shapes of fracture curves, to characterize them in terms of a few basic parameters, to classify the curves according to these parameters, and to indicate the physical meaning of the parameters.

THEORETICAL PART

Equation of shape of the fracture curve

The equation of the fracture curve can be derived on the following assumptions: (1) the fractures propagate from two points isotropically in planes perpendicular to the direction of the acting force, (2) the velocities of fracture propagation are independent of time.

The scheme of formation of a fracture curve can be seen in Figure 1. Let us assume that the primary fracture spreads from the point P at a velocity v_1 and that the secondary fracture starts spreading from the point S with a time lag t_0 at a velocity v_2. At a small difference, h (order of magnitude of microns), between the planes of the primary and secondary fractures (Figure 1a), the material is chipped out in the surroundings of the sites where the ground plans of the fracture fronts meet, and the projection formed on the fracture surface gives rise to a fracture curve, f (Figure 1b).

Besides the mechanism of mutual interaction of two fracture fronts, the origin of fracture curves can also be explained in terms of an interaction of the primary fracture front with the stress wave propagating from the point S. The fracture curves produced by both mechanisms can be described mathematically in a common way.

In order to derive the equation of the curve f, we shall express the time dependences of the distances r_1 and r_2 to which the primary and secondary fracture fronts will propagate from the points P and S respectively by

$$r_1 = v_1 (t+t_0), \tag{1}$$

$$r_2 = v_2 t. \tag{2}$$

The fracture curve consists of the points at which both the fracture fronts under investigation arrive simultaneously. At the same time, for each point M of the curve f, the geometrical condition

$$d = r_2 \cos \phi + [r_1^2 - (r_2 \sin \phi)^2]^{\frac{1}{2}} \tag{3}$$

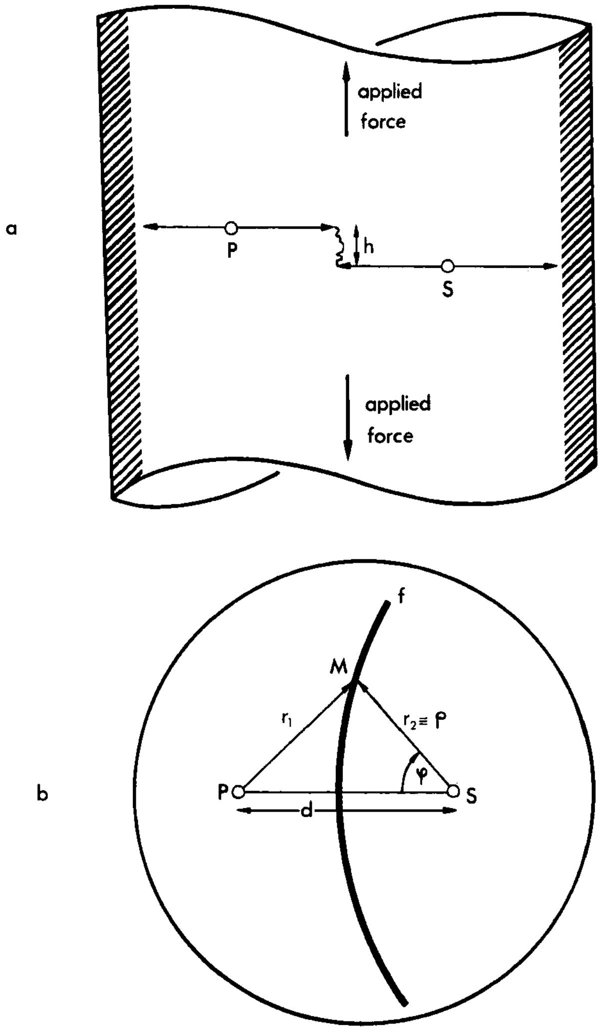


FIGURE 1 Scheme of formation of fracture curves (a) in a plane determined by points P,S and by the direction of the acting force; (b) in a plane perpendicular to the direction of the acting force. P is centre of primary fracture, S is centre of secondary fracture, f is fracture curve formed. Meaning of the other symbols can be seen from the text.

is valid, involving quantities shown in Figure 1b. On introducing the polar coordinates $\rho \equiv r_2$, ϕ (Figure 1b) and ruling out the parameters t and r_1 from Eqs. (1)–(3), we obtain the equation of the fracture curve:

$$\rho^2 [1 - (v_1^2/v_2^2)] - 2\rho [(v_1^2/v_2)\, t_0 + d \cos \phi] - v_1^2 t_0^2 + d^2 = 0. \quad (4)$$

To simplify the equation and analyze it, we introduce the parameters V and a defined as follows:

$$V = v_1/v_2,$$
$$a = d - v_1 t_0. \quad (5)$$

The parameter a shall be designated as the "activation distance", since it represents the distance of the front of the primary fracture from the centre of the secondary fracture S at which the secondary fracture becomes activated (i.e. at the moment $t = 0$, as follows from Eqs. (1) and (2)). On substitution the equation becomes

$$\rho^2 (1 - V^2)/d - 2\rho [V(1 - (a/d)) + \cos \phi] + a [2 - (a/d)] = 0. \quad (6)$$

Classification of the fracture curves

The form of the fracture curves according to Eq. (6) depends on the values of the three parameters d, V, a, of which d can be measured directly, and the other two can be calculated from the form of the fracture curve. We shall now investigate in which ranges of values the parameters have a physical meaning.

The distance between the fracture centers, d, is in fact limited by the dimensions of the sample ($0 \leqq d < l$, l being the longest possible distance on the fracture surface). For the subsequent mathematical analysis of the fracture curves, an approximate case $d \to \infty$, i.e. $1/d = 0$, can also be used, if the highest value of the coordinate ρ of the curve is very low compared to the distance d. A solution to the trivial case $d = 0$ is represented by a circle with its center in the fracture center (P $\equiv$ S). We shall therefore assume in our further considerations that d can assume all positive values.

The velocities ratio, V, has always a finite positive value. The trivial cases $V = 0$ and $V \to \infty$ ($v_1 = 0$ or $v_2 = 0$ respectively) do not give any solution of the shape of the fracture curve except for points P or S respectively.

The region of the values of the activation distance a is limited from above by the distance between the fracture centers, d, and from below by the condition which must be met for the fracture curve to be formed still on the fracture surface. The a values lie within the limits

$$(V - 1)(l - d) < a \leqq d.$$

We shall divide the fracture curves into four groups according to the magnitude of the activation distance a (Figure 2). A survey of the types

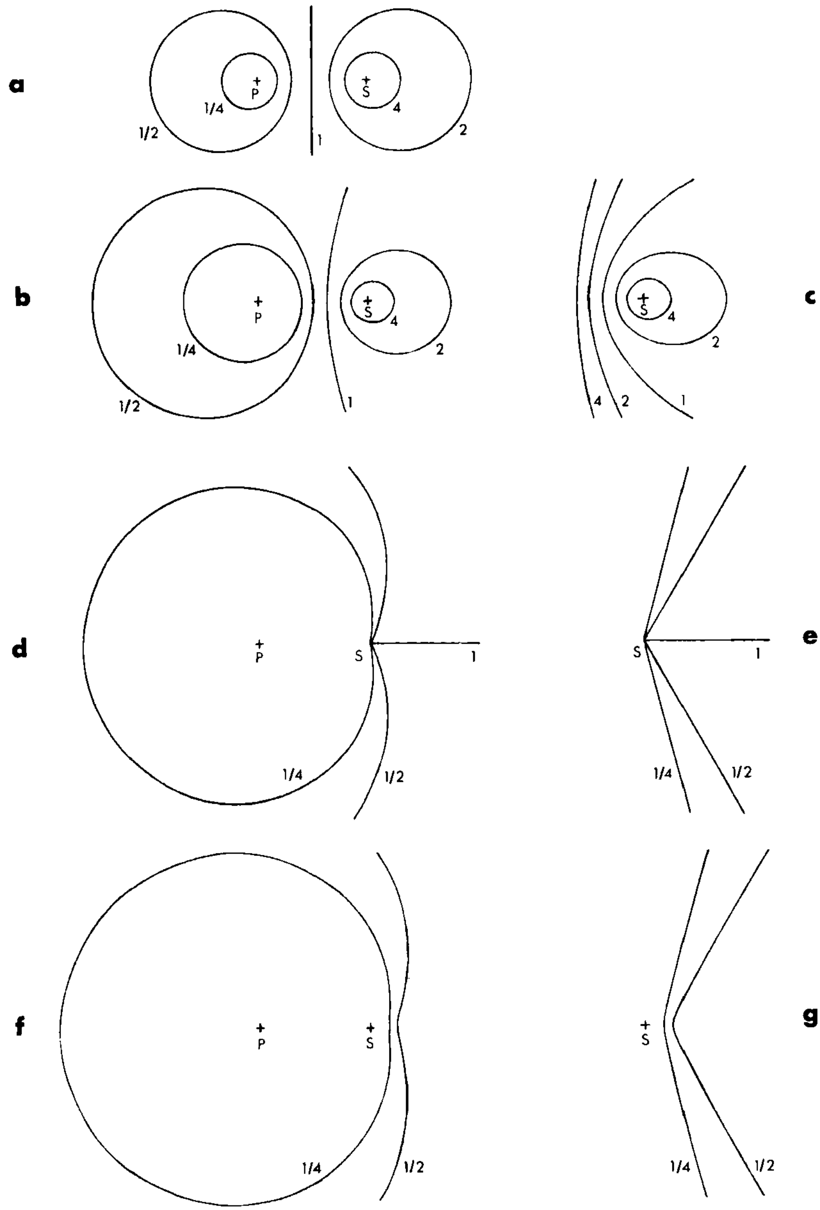


FIGURE 2 Theoretical shapes of fracture curves in the individual groups (cf. text) for $1/d \neq 0$ ($a/d = 3/4$, Figure 2a,b,d,f) and for $1/d = 0$ (Figure 2c,e,g). Group I ($a = d$)—Figure 2a; group II ($0 < a < d$)—Figure 2b,c; group III ($a = 0$)—Figure 2d,e; group IV ($a < 0$)—Figure 2f,g. Numbers at the curves designate value of parameter V.

E

and properties of the fracture curves in the individual groups with respect to the parameters V and d is given in Table I.

TABLE I
Classification of fracture curves

Group	Characteristic of the group	$1/d$	$V < 1$	$V = 1$	$V > 1$
I	$a = d$	$\neq 0$	C_P	L_1	C_S
		0	—	—	—
II	$0 < a < d$	$\neq 0$	R_P	H	R_S
		0†	H	Q	E
III	$a = 0$	$\neq 0$	W	L_2	—
		0	L_3	L_2	—
IV	$a < 0$	$\neq 0$	R_{PS}	—	—
		0	H	—	—

Capital letters designate type of fracture curve: R, closed curve; C, circle; H, hyperbola; Q, parabola; E, ellipse; L, straight line; W, the Wallner line. Indexes P,S designate the fracture centers around which the curve is closed; numerical indexes indicate the type of the straight lines (1, symmetry axis of the intercept PS; 2, semistraight line $\phi = \pi$; 3, two semistraight lines $\phi = \pm$ const.). Other symbols: † $a/d = 0$ is valid simultaneously; — no solution.

Group I This group (Figure 2a) includes fracture curves which are formed with the simultaneous activation of fracture in the primary and secondary centers, i.e. it holds $a = d$.

For this case, Eq. (6) becomes

$$\rho^2 (1 - V^2)/d - 2\rho \cos \phi + d = 0. \tag{7}$$

Equation (7) has a final solution only for $1/d \neq 0$.

For $V \neq 1$ it is a circle, as can be seen from the expression of Eq. (7) in the cartesian coordinates

$$(1 - V^2)(x^2 + y^2) - 2dx + d^2 = 0. \tag{8}$$

The center of the circle lies in the point $(d/(1 - V^2), 0)$; the radius is $Vd/(1 - V^2)$. At $V < 1$, non-concentric circles are formed around the point P, and at $V > 1$, around the point S; the radius of the circle increases with increasing distance between the centers, d, and with the velocities ratio V approaching unity. At the same time as the radius increases, the centers of the circle move away from the points P or S outside their connecting line.

For $V = 1$, we obtain from Eq. (8) the straight line $x = d/2$.

Group II This group (Figures 2b,c, 3) includes fracture curves obtained during activation of the secondary fracture before the front of the primary fracture; that is, it holds $0 < a < d$.

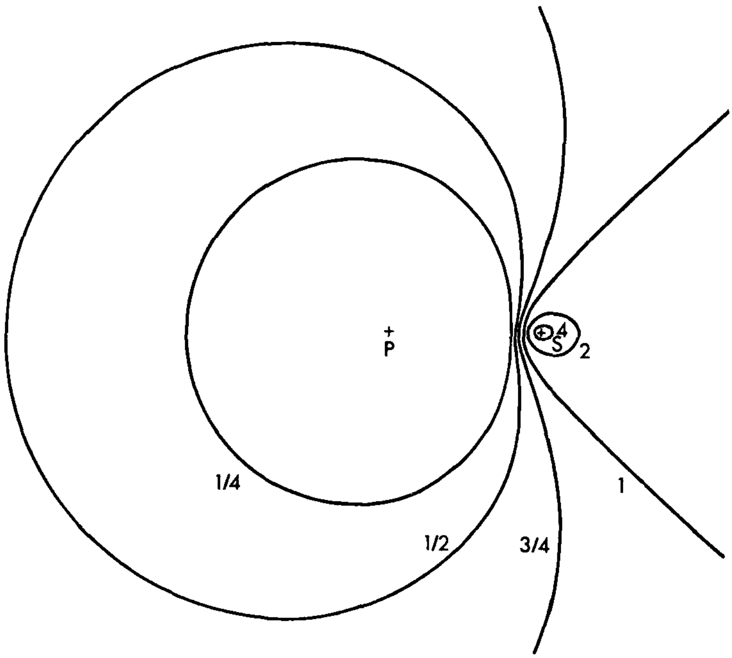


FIGURE 3 Theoretical shape of fracture curve of group II for $a/d = 1/4$.

The general shape of the curves expressed in terms of Eq. (6) can be simplified only if $1/d = 0$ or $V = 1$.

1. For $1/d \neq 0$ and $V = 1$ (Figure 2b) Eq. (6) has the form

$$2\rho\,[1 - (a/d) + \cos\phi] - a\,[2 - (a/d)] = 0. \tag{9}$$

From representation in the cartesian coordinates

$$- 4\,x^2\,(a/d)\,[2 - (a/d)] + 4\,y^2\,[1 - (a/d)]^2 + 4\,xa\,[2 - (a/d)] - a^2\,[2 - (a/d)]^2 = 0 \tag{10}$$

it is evident that in this case the fracture curve is a hyperbola.

2. For $1/d = 0$, and simultaneously $a/d = 0$ (Figure 2c) Eq. (6) is simplified and becomes

$$\rho\,(V + \cos\phi) - a = 0. \tag{11}$$

Equation (11) is a conic section, as can be seen from its representation in the cartesian coordinates

$$x^2 (V^2 - 1) + y^2 V^2 + 2xa - a^2 = 0. \tag{12}$$

The curves at $V < 1$, $V = 1$, and $V > 1$ are represented respectively by a hyperbola, a parabola, and an ellipse.

Group III This group (Figure 2d,e) includes fracture curves formed if the secondary fracture front is activated in the place where the primary fracture front is passing; thus, it holds $a = 0$.

Equation (6) can in this case be written as

$$\rho (1 - V^2)/d - 2 (V + \cos \phi) = 0. \tag{13}$$

Equation (13) has a solution only for $V \leqq 1$. For $V = 1$, the solution is represented by a semi-straight line, $\phi = \pi$.

1. For $1/d \neq 0$ and $V < 1$, the solution of Eq. (13) is represented by a conchoid of the circle (Pascal's spiral) closed around the point P (Figure 2d). This is a common case of the Wallner lines. From the angle of the tangent line to the fracture curve in the point S, the velocities ratio of the fracture propagation, V, can be determined by

$$\lim_{\rho \to 0} \cos \phi = - V. \tag{14}$$

2. For $1/d = 0$, fracture lines are represented by straight lines (Figure 2e) described by the equation

$$V + \cos \phi = 0; \tag{15}$$

in such a case, the parameter V can be determined directly from the equation.

Group IV This group (Figure 2f,g) contains fracture curves which arise during activation of the secondary fracture behind the front of the primary fracture, i.e., $a < 0$.

For this case, Eq. (6) can be solved only for $V < 1$.

1. For $1/d \neq 0$, we obtain a fracture curve closed around both the point P and S (Figure 2f).

2. For $1/d = 0$, Eq. (6) can be simplified:

$$\rho (V + \cos \phi) - a = 0. \tag{16}$$

Equation (16) expresses one branch of the hyperbola (Figure 2g).

Evaluation of parameters of fracture curves

An investigation of the fracture surfaces has as its aim to give data on the fracture mechanism or on the material structure from the morphology of the fracture surface. The fracture curves that appear on the fracture surface can also be an important source of new findings.

According to the extent of information which can be read from the fracture surface about the fracture curves, the region of the parameters V, a can be evaluated qualitatively, or these parameters can be calculated directly by using Eq. (6). To obtain qualitative limits for the parameters V, a from the data on the fracture curves, a system given in Table II has been set up from Table I and Figure 2.

A rough estimate of the parameters is feasible already by using the common form of the fracture curve. If the direction can be determined on the fracture surface in which the crack propagates in the material, the region of the parameter values can be defined with still more accuracy from the orientation of the fracture curve with respect to this direction. If, moreover, also the position of one of the fracture centers is known, it is possible in most cases to assign the fracture curve to the respective group with the narrowest possible delimitation of the parameters V, a. However, in some cases (cf. Table II) the position of both fracture centers must be known for this purpose.

If there is a possibility to determine quantitatively the position of the centers of the primary and secondary fracture, P or S, respectively, the fracture curves allow a quantitative determination of both parameters V and a by the following methods:

1st method—from the shape of the fracture curves; this is a general method that can be applied to all types of fracture curves. The polar coordinates of the fracture curve are determined from the micrograph, and the parameters of the curve are determined from these coordinates by computing non-linear regression. The parameters can be comparatively easily computed with a small-size computer (e.g., Hewlett–Packard).

2nd method—from the diameters of the fracture curves; this method is suited for closed fracture curves only. The parameters a and V can be determined from the coordinates of the fracture curve in the direction of the connecting line PS ($\phi = 0$ or $\phi = \pi$, cf. Figure 1) by using Eq. (6).

(*a*) *Closed curve around the point* P If ρ_{01} and ρ_{02} are coordinates of the points of the curve for which it holds that $\phi = 0$, we have

$$V = (\rho_{01} + \rho_{02} - 2d)/(\rho_{01} - \rho_{02}),$$
$$a = 2\rho_{02}(\rho_{01} - d)/(\rho_{01} - \rho_{02}). \tag{17}$$

(*b*) *Closed curve around the point* S If ρ_0 and ρ_π are coordinates of the points for which $\phi = 0$ or $\phi = \pi$ respectively, it holds

TABLE II

Delimination of the parameter values V, a from data on fracture curves

e	Curve shape	Orientation of curve and position of one center with respect to direction of fracture	Shape of primary fracture front
	Straight or semistraight line, $V \leqq 1, a = 0$ or $a = d$	Perpendicular to direction of fracture, $V = 1, a = d$ Parallel to direction of fracture, $V = 1, a = 0$ Forming an angle with direction of fracture, $V < 1, a = 0$	
	Conic section, $V \leqq 1, a \leqq d, a \neq 0$	Center behind curve, $V \leqq 1, 0 < a < d$	Circular $(1/d \neq 0)$, $V = 1, 0 < a < d$ Straight $(1/d = 0)$, $V \leqq 1, 0 < a < d$
		Center before curve, $V \leqq 1, a < d, a \neq 0$	Circular $(1/d \neq 0)$, $V = 1, 0 < a < d$ Straight $(1/d = 0)$, $V < 1, a < 0$
	Circle, $V \neq 1, a = d$	Nearest point of curve behind, center lying inside curve: $V < 1, a = d$ Nearest point of curve before, center lying inside curve: $V > 1, a = d$	
	Circle-like curve, $V \neq 1, 0 < a < d$	Nearest point of curve behind, center lying inside curve: $V < 1, 0 < a < d$ Nearest point of curve before, center lying inside curve: $V > 1, 0 < a < d$	Circular $(1/d \neq 0)$, $V > 1, 0 < a < d$ Straight $(1/d = 0)$, $V > 1, 0 < a < d$
		Longer diameter parallel to direction of fracture, $V > 1, 0 < a < d$	
	Cardioid-like curve, $V < 1, a < d$	Center lies outside curve, $V < 1, 0 < a < d$ Center lies on curve, $V < 1, a = 0$ Center lies inside curve, $V < 1, a < d$	

$$V = (\rho_\pi + \rho_0)/(\rho_\pi - \rho_0),$$
$$a = 2\,\rho_\pi\,\rho_0/(\rho_\pi - \rho_0). \quad (18)$$

(*c*) *Closed curve around the points* P *and* S If ρ_0 and ρ_π are coordinates of the points for which $\phi = 0$ or $\phi = \pi$ respectively, it holds

$$V = 1 - 2d/(\rho_0 - \rho_\pi),$$
$$a = 2d\,\rho_\pi/(\rho_\pi - \rho_0). \quad (19)$$

At the same time, the agreement or disagreement of results determined by both methods can inform us to what extent the assumptions are fulfilled which were used for describing the real fracture curve by Eq. (6).

EXPERIMENTAL PART

In order to correlate the experimental and theoretical forms of the fracture curves, fracture surfaces of various brittle amorphous materials (polystyrene, poly(methyl methacrylate), copolymers of methyl methacrylate and alyll methacrylate, poly(hydroxyethyl methacrylate), inorganic glass) were obtained by tensile fracture at room temperature. The micrographs of aluminium-coated fracture surfaces (Figures 4 and 5) indicate the existence of most varied forms of fracture curves. Criteria summarized in Table II allows to delimit the region of the values of the parameters V and a. The parameters of some fracture curves were evaluated quantitatively from their shapes (1st method—chapter 2). The values thus obtained together with those of the evaluation of other fracture curves taken from the literature are summarized in Table III.

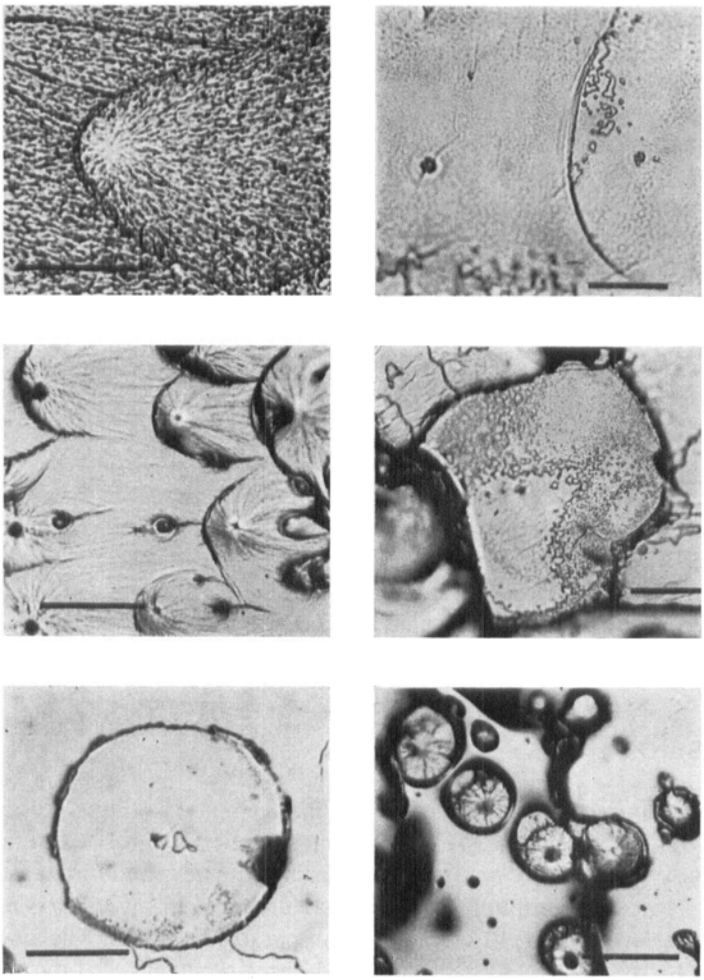
For some selected closed curves (Figure 5) the parameters were evaluated quantitatively by both methods, that is, from the forms and diameters of the curves (Table IV).

DISCUSSION

Comparatively very little attention has so far been paid to the solution of the shape of the fracture curves.[2–4] The existing analytical expressions of fracture curves have been based on the cartesian coordinates, and for reasons of simplicity special assumptions as to the fracture mechanism have been introduced; they have been restricted by certain parameter values. The analytical solutions for some shapes of the curves[2–4,7] are special cases of our analysis based on Eq. (6).

The shape of the fracture curve thus obtained depends on the parameters d, V, a. The individual shapes of the curves can be seen in Figure 2, which at the same time adequately demonstrates the transition of their shapes with a change in the parameter and between the groups, with the exception of the transition from group II to group III. The change in the sign of curvature in the point ρ_0 for $V < 1$ is obvious at a lower value of a/d from Figure 3.

A qualitative comparison of the fracture curves observed (Figures 4 and 5) with those derived (Figure 2) indicates that the majority of the theoretically



a b c d e f

FIGURE 4a–f.

predicted shapes can actually be detected on the fracture surfaces. Open and closed curves of group II are those most frequently found (Figure 4a–c,f); circles have also been found (Figure 4e), as well as intercepts of straight lines at the points of contact of the fracture formations (Figure 5c) characteristic of group I of the fracture curves. Curves of group III have been observed for both polymeric materials and inorganic glass: the Wallner lines (Figure 4j,k) and straight lines (Figure 4g). A cardioid-like curve (Figure 4d,h) has also been found; it can belong to group II, III, or IV. Identification of this curve would require an unambiguous determination of the position of the fracture centres (cf. Table II).

The fulfilment of assumptions used for the description of the fracture curves by means of Eq. (6) can be established by verifying that both the initial and the final part of the fracture curve is formed under the same conditions. To test this finding, a comparison must be made of the parameter values of curves calculated by the first method (from the shape of the curve) applied

g h

j k

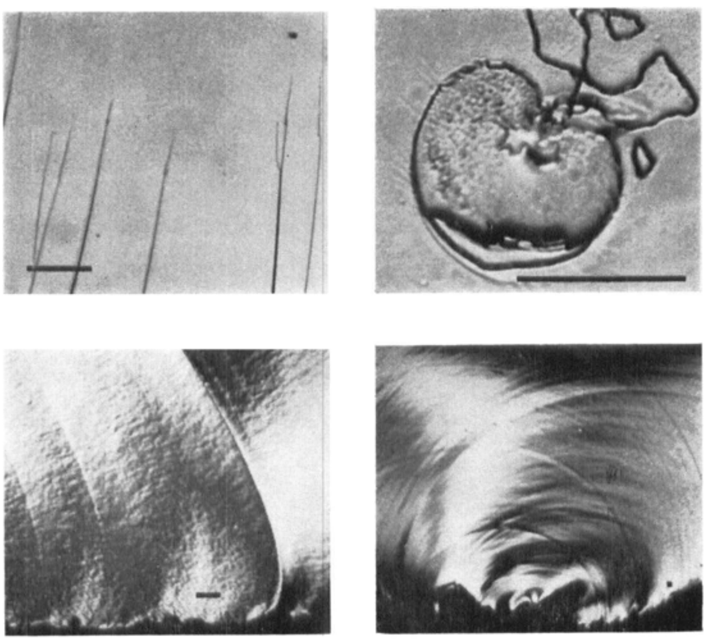


FIGURE 4g–k.

FIGURE 4 Shapes of fracture curves on fracture surfaces of brittle amorphous materials. a, poly(methyl methacrylate); b–f, polystyrene; g, poly(hydroxyethyl methacrylate); h, polystyrene; j, copolymer of methyl methacrylate and alyll methacıylate; k, inorganic glass. The bar scale represents 25 μm.

to a part of the fracture curve adjacent to the primary fracture, and also by the second method (from the diameter) which involves the beginning and the end of the curve. In this way, curves from Figure 5 (Table IV) were evaluated. The shape of curve 1 given in Figure 5a resembles the shape which is theoretically possible; Table IV also shows a very good agreement of the parameter values. Curves 2,3, Figure 5b,c reveal at the first glance the difference of the shapes from those theoretically predicted; the parameter values calculated by both methods also differ considerably. It can be calculated from the shape of parts of the curves around S adjacent to the primary centre P that the curve should theoretically be closed at a shorter (curve 2, Figure 5b) or a longer (curve 3, Figure 5c) distance from the fracture centres than the actual

a

b

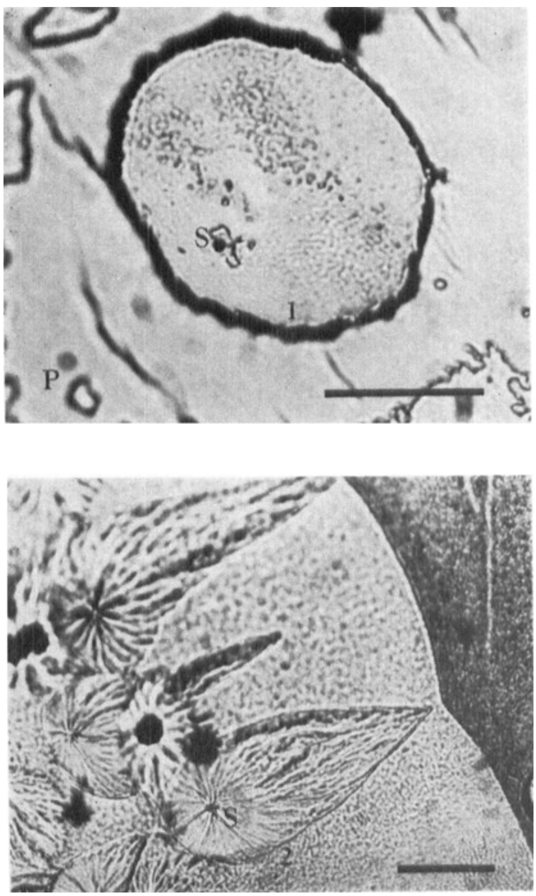


FIGURE 5a–b.

distance. This fact can be explained, in the first case, either by a sudden slowing-down of the primary fracture front or by an acceleration of the secondary front; in the second case, the explanation should probably be sought in an acceleration of the primary fracture.

This phenomenon of a sudden change in the velocity of propagation of the fracture surfaces has been observed in some cases only. In most cases, the shapes of the fracture curves resemble those theoretically derived, which together with other data found in the literature[12] indicates that the assumption of the propagation of fracture fronts at constant velocities may be justified.

The knowledge of the parameter values of the fracture curves can give important information about the mechanism of the fracture formation. As yet, this has been done by following ρ_0 depending on the distance of the respective fracture curve starting with the beginning from which the crack propagates in the material.[11] However, ρ_0 has no unambiguous physical interpretation (cf. Eqs. (17)–(19)). We believe that a more exact picture about the fracture propagation is offered by the parameters V, a, d, each of which can characterize a certain feature of the fracture process. The parameter d is proportional to the density of the original and fracture-initiated defects in the sample, while the activation distance a indicates the critical character of the

c

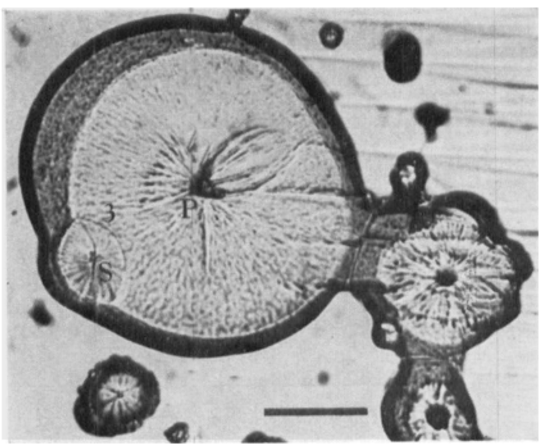


FIGURE 5c.

FIGURE 5 Closed curves whose parameters are evaluated by two methods (fracture surface of polystyrene). Evaluated parameters of curves are given in Table IV. The bar scale represents 25 μm.

TABLE III

Parameters of experimental fracture curves

V	a(μm)	V	a(μm)
Poly(methyl methacrylate)		Polystyrene	
1.18	43.2	1.65[a]	24.0[a]
1.14	26.0	1.62[b]	51.2[b]
1.10	27.5	1.15[c]	0.89[c]
1.07	32.1		
1.04	11.9	Glass	
1.03	34.4	0.65[d]	≈0[d]
0.93	20.5	0.27[e]	≈0[e]

[a] Curve 1, Figure 5a; [b] Figure 4b; [c] Ref. 8, Figure 10; [d] Ref. 6, Figure 2; [e] Figure 4k.

TABLE IV

Comparison of evaluation results of closed fracture curves obtained by two methods

Parameter	1st method—from curve shape	2nd method—from curve diameter
	Curve 1 (Figure 5a)	
V	1.3	1.6
a(μm)	19	21
a/d	0.61	0.68
	Curve 2 (Figure 5b)	
V	4.0	1.5
a(μm)	62	32
a/d	0.06	0.03
	Curve 3 (Figure 5c)	
V	2.2	7.1
a(μm)	26	54
a/d	0.7	1.5

defects. The velocities ratio V and the values of the other parameters of the subsequent fractures whose centres are situated on a common straight line PS allow to draw a conclusion about the change in the velocity of propagation of the fracture crack.

It follows from the established values of the parameters V, a from the selected fracture curves of various materials (Table III) that the values a are positive, and approximately zero in the case of the Wallner lines. It is noteworthy that with increasing hardness of the materials (in the order polystyrene, poly(methyl methacrylate), glass), V on the average decreases. This phenomenon is probably connected with the different rate of relaxation of the material which appears during healing of the secondary fraction. However, a detailed explanation including the relationships between the parameters of the curves, on the one hand, and the material constants and the fracture conditions, on the other, calls for further investigation.

References

1. J. P. Berry, *J. Appl. Phys.* **33,** 1741 (1962).
2. E. H. Andrews, *Fracture in Polymers*, Oliver & Boyd, Edinburgh and London, 1968.
3. F. Schwazl, A. J. Staverman, *Die Physik der Hochpolymeren*, bd. IV, H. A. Stuart, Ed., Springer, Berlin, 1956.
4. V. R. Regel, *Z. Tekh. Phys.* **26,** 359 (1956).
5. H. Wallner, *Z. Phys.* **114,** 368 (1939).
6. A. G. Smekal, *Acta Phys. Austr.* **7,** 110 (1953).
7. E. H. Andrews, *J. Appl. Phys.* **30,** 740 (1959).
8. R. J. Bird, G. Rooney, and J. Mann, *Polymer* **12,** 742 (1971).
9. R. J. Bird, J. Mann, G. Pogany, and G. Rooney, *Polymer* **7,** 307 (1966).
10. J. Murray and D. Hull, *J. Polymer Sci.* A-2, **8,** 1521 (1970).
11. Ir. J. Leeuwerik, *Rheologica Acta* **2,** 10 (1962).
12. H. Schardin, *Fracture*, p. 297, B. L. Averbach, D. K. Felbeck, G. T. Hahn, and D. A. Thomas, Eds., Wiley, N.Y., 1959.